PERGAMON

International Journal of Heat and Mass Transfer 45 (2002) 4643–4653

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

# An inverse problem in simultaneous estimating the Biot numbers of heat and moisture transfer for a porous material

Cheng-Hung Huang *, Chun-Ying Yeh

*Department of Naval Architecture and Marine Engineering, National Cheng Kung University, Tainan 701, Taiwan, ROC*

Received 16 November 2001; received in revised form 16 April 2002

**Abstract**

A conjugate gradient method based inverse algorithm is applied in the present study in simultaneous determining the unknown time-dependent Biot numbers of heat and moisture transfer for a porous material based on interior measurements of temperature and moisture.

It is assumed that no prior information is available on the functional form of the unknown Biot numbers in the present study, thus, it is classified as the function estimation in inverse calculation.

The accuracy of this inverse heat and moisture transfer problem is examined by using the simulated exact and inexact temperature and moisture measurements in the numerical experiments. Results show that the estimation on the time-dependent Biot numbers can be obtained with any arbitrary initial guesses on a Pentium IV 1.4 GHz personal computer.



## 1. Introduction

The direct heat and moisture transfer problem is concerned with the determination of temperature and moisture at interior points of a region when the initial and boundary conditions and thermophysical properties are specified. In contrast, the inverse heat and moisture transfer problem considered here involves the determination of the unknown time-dependent Biot numbers for heat and moisture transfer in a porous material from the knowledge of the temperature and moisture measurements taken within the body.

For the conventional inverse heat transfer problems, the estimation of unknown thermal boundary conditions are always the main concerns. The technique of conjugate gradient method (CGM) [1] has been shown its potential for these kind of problems and has been applied to many applications. For instant Huang and Chen [2] used boundary element method and CGM to estimate the boundary heat fluxes for an irregular domain. Huang and Wang [3] used CGM in estimating surface heat fluxes for a three-dimensional inverse heat conduction problem. Huang and Chen [4] used same technique in estimating surface heat fluxes for a three-dimensional inverse heat convection problem. However, the inverse problem for coupled heat and moisture transport is very limited in the literature.

Recently, Chang and Weng [5] used similar algorithm to determine the moisture flux for an inverse heat and moisture transfer problem based on temperature measurements. In their paper the boundary conditions are assumed in a very ideal situation, i.e. they did not apply energy and mass balance at the boundaries. This will certainly simplify the inverse analysis but at the same time may also not realistic for the applications. For the case when internal measurement is applied, a two-layer problem is needed to obtain the analytical for the adjoint problem. Moreover, there is a suspicious result in their paper that will be discussed later in text.

For this reason the present paper is to extend their study with some improvements. Firstly, the original

* Corresponding author. Tel.: +886-6-274-7018; fax: +886-6-274-7019.

*E-mail address:* chhuang@mail.ncku.edu.tw (C.-H. Huang).

0017-9310/02/$ - see front matter © 2002 Published by Elsevier Science Ltd.
PII: S0017-9310(02)00161-8

**Nomenclature**

| | |
|---|---|
| $Bi_m(\tau)$ | Biot number for mass transfer |
| $Bi_q(\tau)$ | Biot number for heat transfer |
| $J$ | functional defined by Eq. (2) |
| $J_1'$, $J_2'$ | gradient of functional defined by Eqs. (16) and (17) |
| $Ko$ | Kossovitch number |
| $Lu$ | Luikov number |
| $P_1$, $P_2$ | direction of descent defined by Eqs. (4a) and (4b) |
| $Pn$ | Possnov number |
| $Q$ | dimensional heat flux |
| $X$ | dimensionless coordinate |
| $Y_1(X,\tau)$ | measured dimensionless temperature |
| $Y_2(X,\tau)$ | measured dimensionless moisture |
| *Greek symbols* | |
| $\tau$ | dimensionless time |
| $\beta_1$, $\beta_2$ | search step sizes |
| $\gamma_1$, $\gamma_2$ | conjugate coefficients |
| $\delta(\bullet)$ | Dirac delta function |
| $\lambda_1(X,\tau)$, $\lambda_2(X,\tau)$ | Lagrange multiplier defined by Eqs. (13a)–(13h) |
| $\theta_1(X,\tau)$ | estimated dimensionless temperature |
| $\theta_2(X,\tau)$ | estimated dimensionless moisture |
| $\Delta\bar{\theta}_1$, $\Delta\bar{\theta}_2$, $\Delta\tilde{\theta}_1$ and $\Delta\tilde{\theta}_2$ | sensitivity function defined by Eqs. (6a)–(7h) |
| $\varepsilon$ | phase change criterion |
| $\eta$ | convergence criteria |
| *Superscript* | |
| $n$ | iteration index |

Lukiov's equation [6] that governing the heat and moisture transfer with energy and mass balance at the boundaries are considered and solved in the present inverse analysis. The derivation of the relevant equations for use in CGM is thus more involved but also more realistic. Secondly, two unknown time-dependent Biot numbers of heat and moisture transfer for a porous material are to be estimated simultaneously based on temperature and moisture measurements. For this reason two sensitivity problems and two search step sizes are needed in the present study. An explicit expression for the determination of search step sizes will be derived with the help of the solutions of sensitivity problems in text. Thirdly, we will demonstrate in text that it is not necessary to arrange the inverse problem as a two-layer problem in the present study when interior measurements are used. This fact will simplify the entire inverse analysis.

The CGM derives basis from the perturbational principles [7] and transforms the inverse problem to the solution of three problems, namely, the direct problem, the sensitivity problem and the adjoint problem, which will be discussed in detail in text.

## 2. Direct problem

To illustrate the methodology for developing expressions for use in simultaneous determining two unknown Biot numbers $Bi_q(\tau)$ and $Bi_m(\tau)$ for heat and moisture transfer, respectively, we consider the following inverse problem. A slab of thickness $L$ is initially at temperature $T(x,0) = T_0$ and moisture $u(x,0) = u_0$. For time $t > 0$, the boundary surface at $x = 0$ is subjected to a heat flux $q$, while $x = L$ is subjected to third kind boundary condition for both heat and moisture transfer.

If the following dimensionless quantities are defined [8]

$$Bi_m(\tau) = \frac{h_m L}{k_m},\quad Bi_q(\tau) = \frac{hL}{k},\quad Ko = \frac{r}{c}\frac{u_0 - u^*}{T_s - T_0},$$
$$Lu = \frac{a_m}{a},\quad Pn = \delta\frac{T_s - T_0}{u_0 - u^*},\quad Q = \frac{qL}{k(T_s - T_0)},$$
$$X = \frac{x}{L},\quad \theta_1 = \frac{T - T_0}{T_s - T_0},\quad \theta_2 = \frac{u_0 - u}{u_0 - u^*},\quad \tau = \frac{at}{L^2}.$$

Here $k$ and $k_m$ are the thermal and moisture conductivity, $T_s$ is the temperature of surrounding air, $u^*$ is the moisture in equilibrium with surrounding air, $h$ and $h_m$ are the heat and mass transfer coefficient, $a$ and $a_m$ are the thermal and moisture diffusivity, $r$ is the specific heat of evaporation and $c$ is the specific heat capacity of material. Fig. 1 shows the schematic picture of the present study.

The dimensionless formulation of this heat and moisture transfer problem can be expressed as:

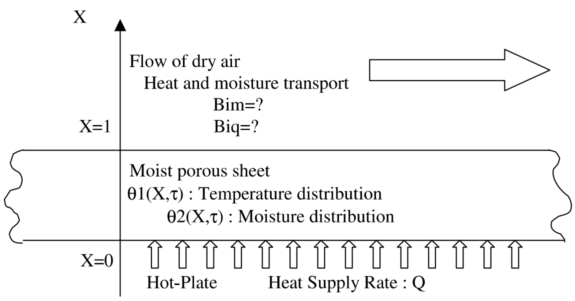


Fig. 1. Schematic picture of the present study.

$$\frac{\partial\theta_1(X,\tau)}{\partial\tau}=\frac{\partial^2\theta_1(X,\tau)}{\partial X^2}-\varepsilon Ko\frac{\partial\theta_2(X,\tau)}{\partial\tau},\quad \text{in } 0\leqslant X\leqslant 1,\ \tau>0, \tag{1a}$$

$$\frac{\partial\theta_2(X,\tau)}{\partial\tau}=Lu\frac{\partial^2\theta_2(X,\tau)}{\partial X^2}-Lu\,Pn\frac{\partial^2\theta_1(X,\tau)}{\partial X^2},\quad \text{in } 0\leqslant X\leqslant,\ \tau>0. \tag{1b}$$

Subjected to the following initial and boundary conditions

$$\theta_1(X,0)=0,\quad \text{in } 0\leqslant X\leqslant 1,\ \tau=0, \tag{1c}$$

$$\theta_2(X,0)=0,\quad \text{in } 0\leqslant X\leqslant 1,\ \tau=0, \tag{1d}$$

$$\frac{\partial\theta_1(X,\tau)}{\partial X}=-Q,\quad \text{at } X=0, \tag{1e}$$

$$\frac{\partial\theta_2(X,\tau)}{\partial X}=-Pn\frac{\partial\theta_1(X,\tau)}{\partial X},\quad \text{at } X=0, \tag{1f}$$

$$\frac{\partial\theta_1(X,\tau)}{\partial X}-Bi_{\mathrm{q}}(\tau)[1-\theta_1]+(1-\varepsilon)Ko\,Lu\,Bi_{\mathrm{m}}(\tau)[1-\theta_2]=0,\quad \text{at } X=1, \tag{1g}$$

$$-\frac{\partial\theta_2(X,\tau)}{\partial X}+Pn\frac{\partial\theta_1(X,\tau)}{\partial X}+Bi_{\mathrm{m}}(\tau)[1-\theta_2]=0,\quad \text{at } X=1. \tag{1h}$$

Here $\varepsilon$ is the phase change criterion. It is obvious that the present problem is coupled both in governing differential equation and in boundary conditions. The direct problem considered here is concerned with calculating the medium temperature and moisture when the Biot numbers $Bi_{\mathrm{q}}(\tau)$ and $Bi_{\mathrm{m}}(\tau)$, thermal properties and initial and boundary condition are known. The Crank–Nicolson finite differences method with iterations can be used to solve this coupled direct problem.

## 3. Inverse problem

For the inverse problem considered here, the Biot numbers $Bi_{\mathrm{q}}(\tau)$ and $Bi_{\mathrm{m}}(\tau)$ are regarded as being unknown, but everything else in Eqs. (1a)–(1h) is known. In addition, the measured temperature and moisture distributions within the space domain at any time are considered available.

Let the measured temperature and moisture at position $X_{\mathrm{s}}$ and time $\tau$ be denoted by $Y_1(X_{\mathrm{s}},\tau)$ and $Y_2(X_{\mathrm{s}},\tau)$, respectively. Then this inverse problem can be stated as follows: by utilizing the above mentioned measured temperature and moisture data $Y_1(X_{\mathrm{s}},\tau)$ and $Y_2(X_{\mathrm{s}},\tau)$, estimate the unknown Biot numbers $Bi_{\mathrm{q}}(\tau)$ and $Bi_{\mathrm{m}}(\tau)$ over the specified time domain.

The solution of the present inverse problem is to be obtained in such a way that the following functional is minimized:

$$\begin{aligned}J\left[Bi_{\mathrm{q}}(\tau),Bi_{\mathrm{m}}(\tau)\right]&=\int_{\tau=0}^{\tau_{\mathrm{f}}}[\theta_1(X_{\mathrm{s}},\tau)-Y_1(X_{\mathrm{s}},\tau)]^2\,\mathrm{d}\tau+\int_{\tau=0}^{\tau_{\mathrm{f}}}[\theta_2(X_{\mathrm{s}},\tau)-Y_2(X_{\mathrm{s}},\tau)]^2\,\mathrm{d}\tau\\&=\int_{X=0}^{1}\int_{\tau=0}^{\tau_{\mathrm{f}}}[\theta_1(X,\tau)-Y_1(X,\tau)]^2\delta(X-X_{\mathrm{s}})\,\mathrm{d}\tau\,\mathrm{d}X\\&\quad+\int_{X=0}^{1}\int_{\tau=0}^{\tau_{\mathrm{f}}}[\theta_2(X,\tau)-Y_2(X,\tau)]^2\delta(X-X_{\mathrm{s}})\,\mathrm{d}\tau\,\mathrm{d}X.\end{aligned} \tag{2}$$

Here $\theta_1(X,\tau)$ and $\theta_2(X,\tau)$ are the estimated (or computed) temperature and moisture at time $\tau$. These quantities are determined from the solution of the direct problem given previously by using the estimated Biot numbers $Bi_{\mathrm{q}}(\tau)$ and $Bi_{\mathrm{m}}(\tau)$. $\delta(\bullet)$ is the Dirac delta function.

## 4. Conjugate gradient method for minimization

The following iterative process based on the CGM [1] is now used for the estimation of Biot numbers $Bi_{\mathrm{q}}(\tau)$ and $Bi_{\mathrm{m}}(\tau)$ by minimizing the above functional $J[Bi_{\mathrm{q}}(\tau),Bi_{\mathrm{m}}(\tau)]$:

$$Bi_{\mathrm{q}}^{n+1}(\tau)=Bi_{\mathrm{q}}^{n}(\tau)-\beta_1^nP_1^n(\tau),\quad n=0,1,2,\ldots, \tag{3a}$$

$$Bi_{\mathrm{m}}^{n+1}(\tau)=Bi_{\mathrm{m}}^{n}(\tau)-\beta_2^nP_2^n(\tau),\quad n=0,1,2,\ldots, \tag{3b}$$

where $\beta_1^n$ and $\beta_2^n$ are the search step sizes in going from iteration $n$ to iteration $n+1$, and $P_1^n(\tau)$ and $P_2^n(\tau)$ are the directions of descent (i.e. search directions) given by

$$P_1^n(\tau)=J_1'^n(\tau)+\gamma_1^nP_1^{n-1}(\tau), \tag{4a}$$

$$P_2^n(\tau)=J_2'^n(\tau)+\gamma_2^nP_2^{n-1}(\tau), \tag{4b}$$

which is a conjugation of the gradient directions $J_1'^n(\tau)$ and $J_2'^n(\tau)$ at iteration $n$ and the directions of descent $P_1^{n-1}(\tau)$ and $P_2^{n-1}(\tau)$ at iteration $n-1$. The conjugate coefficient is determined from

$$\gamma_1^n=\frac{\int_{\tau=0}^{\tau_{\mathrm{f}}}[J_1'^n(\tau)]^2\,\mathrm{d}\tau}{\int_{\tau=0}^{\tau_{\mathrm{f}}}[J_1'^{n-1}(\tau)]^2\,\mathrm{d}\tau},\quad \text{with } \gamma_1^0=0, \tag{5a}$$

$$\gamma_2^n=\frac{\int_{\tau=0}^{\tau_{\mathrm{f}}}[J_2'^n(\tau)]^2\,\mathrm{d}\tau}{\int_{\tau=0}^{\tau_{\mathrm{f}}}[J_2'^{n-1}(\tau)]^2\,\mathrm{d}\tau},\quad \text{with } \gamma_2^0=0. \tag{5b}$$

To perform the iterations according to Eqs. (4a) and (4b), we need to compute the step sizes $\beta_1^n$ and $\beta_2^n$ and the gradient of the functional $J_1'^n(\tau)$ and $J_2'^n(\tau)$. In order to develop expressions for the determination of these two quantities, two sensitivity problems and an adjoint problem are constructed as described below.

## 5. Sensitivity problems and search step sizes

Since the problem involves two unknown Biot numbers $Bi_q(\tau)$ and $Bi_m(\tau)$, in order to derive the sensitivity problem for each unknown function, we should perturb the unknown function one at a time.

Firstly, it is assumed that when $Bi_q(\tau)$ undergoes a variation $\Delta Bi_q(\tau)$, $\theta_1(X,\tau)$ and $\theta_2(X,\tau)$ are perturbed by $\Delta\bar{\theta}_1$ and $\Delta\bar{\theta}_2$. Then replacing in the direct problem $Bi_q$ by $Bi_q+\Delta Bi_q$, $\theta_1$ by $\theta_1+\Delta\bar{\theta}_1$ and $\theta_2$ by $\theta_2+\Delta\bar{\theta}_2$, subtracting from the resulting expressions the direct problem and neglecting the second-order terms, the following sensitivity problem for the sensitivity functions $\Delta\bar{\theta}_1$ and $\Delta\bar{\theta}_2$ are obtained.

$$\frac{\partial\Delta\bar{\theta}_1(X,\tau)}{\partial\tau}=\frac{\partial^2\Delta\bar{\theta}_1(X,\tau)}{\partial X^2}-\varepsilon Ko\frac{\partial\Delta\bar{\theta}_2(X,\tau)}{\partial\tau},\quad \text{in } 0\leqslant X\leqslant 1,\ \tau>0, \tag{6a}$$

$$\frac{\partial\Delta\bar{\theta}_2(X,\tau)}{\partial\tau}=Lu\frac{\partial^2\Delta\bar{\theta}_2(X,\tau)}{\partial X^2}-Lu\,Pn\frac{\partial^2\Delta\bar{\theta}_1(X,\tau)}{\partial X^2},\quad \text{in } 0\leqslant X\leqslant 1,\ \tau>0. \tag{6b}$$

Subjected to the following initial and boundary conditions

$$\Delta\bar{\theta}_1(X,0)=0,\quad \text{in } 0\leqslant X\leqslant 1,\ \tau>0, \tag{6c}$$

$$\Delta\bar{\theta}_2(X,0)=0,\quad \text{in } 0\leqslant X\leqslant 1,\ \tau>0, \tag{6d}$$

$$\frac{\partial\Delta\bar{\theta}_1(X,\tau)}{\partial X}=0,\quad \text{at } X=0, \tag{6e}$$

$$\frac{\partial\Delta\bar{\theta}_2(X,\tau)}{\partial X}=-Pn\frac{\partial\Delta\bar{\theta}_1(X,\tau)}{\partial X},\quad \text{at } X=0, \tag{6f}$$

$$\frac{\partial\Delta\bar{\theta}_1(X,\tau)}{\partial X}-\Delta Bi_q(\tau)[1-\theta_1]+Bi_q(\tau)\Delta\bar{\theta}_1-(1-\varepsilon)Ko\,Lu\,Bi_m(\tau)\Delta\bar{\theta}_2=0,\quad \text{at } X=1, \tag{6g}$$

$$-\frac{\partial\Delta\bar{\theta}_2(X,\tau)}{\partial X}+Pn\frac{\partial\Delta\bar{\theta}_1(X,\tau)}{\partial X}-Bi_m(\tau)\Delta\bar{\theta}_2=0,\quad \text{at } X=1. \tag{6h}$$

Similarly, by perturbing $Bi_m(\tau)$ with $\Delta Bi_m(\tau)$, the second sensitivity problem can be obtained as

$$\frac{\partial\Delta\tilde{\theta}_1(X,\tau)}{\partial\tau}=\frac{\partial^2\Delta\tilde{\theta}_1(X,\tau)}{\partial X^2}-\varepsilon Ko\frac{\partial\Delta\tilde{\theta}_2(X,\tau)}{\partial\tau},\quad \text{in } 0\leqslant X\leqslant 1,\ \tau>0, \tag{7a}$$

$$\frac{\partial\Delta\tilde{\theta}_2(X,\tau)}{\partial\tau}=Lu\frac{\partial^2\Delta\tilde{\theta}_2(X,\tau)}{\partial X^2}-Lu\,Pn\frac{\partial^2\Delta\tilde{\theta}_1(X,\tau)}{\partial X^2},\quad \text{in } 0\leqslant X\leqslant 1,\ \tau>0. \tag{7b}$$

Subjected to the following initial and boundary conditions

$$\Delta\tilde{\theta}_1(X,0)=0,\quad \text{in } 0\leqslant X\leqslant 1,\ \tau=0, \tag{7c}$$

$$\Delta\tilde{\theta}_2(X,0)=0,\quad \text{in } 0\leqslant X\leqslant 1,\ \tau=0, \tag{7d}$$

$$\frac{\partial\Delta\tilde{\theta}_1(X,\tau)}{\partial X}=0,\quad \text{at } X=0, \tag{7e}$$

$$\frac{\partial\Delta\tilde{\theta}_2(X,\tau)}{\partial X}=-Pn\frac{\partial\Delta\tilde{\theta}_1(X,\tau)}{\partial X},\quad \text{at } X=0, \tag{7f}$$

$$\frac{\partial\Delta\tilde{\theta}_1(X,\tau)}{\partial \mathrm{X}}+Bi_q(\tau)\Delta\tilde{\theta}_1-(1-\varepsilon)Ko\,Lu[\Delta Bi_m(\tau)(1-\theta_2)-Bi_m(\tau)\Delta\tilde{\theta}_2]=0,\quad \text{at } X=1, \tag{7g}$$

$$-\frac{\partial\Delta\tilde{\theta}_2(X,\tau)}{\partial \mathrm{X}}+Pn\frac{\partial\Delta\tilde{\theta}_1(X,\tau)}{\partial X}+\Delta Bi_m(\tau)(1-\theta_2)-Bi_m(\tau)\Delta\tilde{\theta}_2=0,\quad \text{at } X=1. \tag{7h}$$

We should note that the above sensitivity problems can also be solved by Crank–Nicolson finite difference method with iterations.

The functional $J[Bi_q(\tau),Bi_m(\tau)]$ for iteration $n+1$ is obtained by rewriting Eq. (2) as

$$J[Bi_q^{n+1}(\tau),Bi_m^{n+1}(\tau)]=\int_{\tau=0}^{\tau}[\theta_1(X_s,\tau;Bi_q^n-\beta_1^nP_1^n,Bi_m^n-\beta_2^nP_2^n)-Y_1(X_s,\tau)]^2\,d\tau+\int_{\tau=0}^{\tau}[\theta_2(X_s,\tau;Bi_q^n-\beta_1^nP_1^n,Bi_m^n-\beta_2^nP_2^n)-Y_2(X_s,\tau)]^2\,d\tau, \tag{8}$$

where we replaced $Bi_q^{n+1}(\tau)$ and $Bi_m^{n+1}(\tau)$ by the expression given by Eqs. (3a) and (3b).

If the estimated temperatures $\theta_1(X_s,\tau;Bi_q^n-\beta_1^nP_1^n,Bi_m^n-\beta_2^nP_2^n)$ and moistures $\theta_2(X_s,\tau;Bi_q^n-\beta_1^nP_1^n,Bi_m^n-\beta_2^nP_2^n)$ are linearized by a Taylor expansion, Eq. (8) takes the form:

$$J[Bi_q^{n+1}(\tau),Bi_m^{n+1}(\tau)]=\int_{\tau=0}^{\tau_f}[\theta_1(X_s,\tau;Bi_q^n,Bi_m^n)-\beta_1^n\Delta\bar{\theta}_1(P_1^n)-\beta_2^n\Delta\tilde{\theta}_1(P_2^n)-Y_1(X_s,\tau)]^2\,d\tau+\int_{\tau=0}^{\tau_f}[\theta_2(X_s,\tau;Bi_q^n,Bi_m^n)-\beta_1^n\Delta\bar{\theta}_2(P_1^n)-\beta_2^n\Delta\tilde{\theta}_2(P_2^n)-Y_2(X_s,\tau)]^2\,d\tau, \tag{9}$$

where $\theta_1(X_s,\tau;Bi_q^n,Bi_m^n)$ and $\theta_2(X_s,\tau;Bi_q^n,Bi_m^n)$ are the solution of the direct problem by using estimate $Bi_q(\tau)$ and $Bi_m(\tau)$ at time $\tau$.

The sensitivity functions $\Delta\bar{\theta}_1(P_1^n)$, $\Delta\bar{\theta}_2(P_1^n)$ and $\Delta\tilde{\theta}_1(P_2^n)$, $\Delta\tilde{\theta}_2(P_2^n)$ are taken as the solutions of problems (6a)–(7h) at time $\tau$ by letting $\Delta Bi_q(\tau)=P_1^n(\tau)$ in Eq. (6g) and $\Delta Bi_m(\tau)=P_2^n(\tau)$ in Eqs. (7g) and (7h), respectively.

Eq. (9) is differentiated with respect to $\beta_1^n$ and $\beta_2^n$, respectively, and equating them equal to zero to obtain two independent equations. After solving these two equations, the search step sizes $\beta_1^n$ and $\beta_2^n$ can be determined as:

$$\beta_1^n = (C_3C_5 - C_2C_4)/(C_3C_3 - C_1C_2), \tag{10a}$$

$$\beta_2^n = (C_3C_4 - C_1C_5)/(C_3C_3 - C_1C_2), \tag{10b}$$

where

$$C_1 = \int_{\tau=0}^{\tau_f} (\Delta\bar{\theta}_1^2 + \Delta\bar{\theta}_2^2)\,d\tau, \tag{10c}$$

$$C_2 = \int_{\tau=0}^{\tau_f} (\Delta\tilde{\theta}_1^2 + \Delta\tilde{\theta}_2^2)\,d\tau, \tag{10d}$$

$$C_3 = \int_{\tau=0}^{\tau_f} (\Delta\bar{\theta}_1\Delta\tilde{\theta}_1 + \Delta\bar{\theta}_2\Delta\tilde{\theta}_2)\,d\tau, \tag{10e}$$

$$C_4 = \int_{\tau=0}^{\tau_f} [(\theta_1 - Y_1)\Delta\bar{\theta}_1 + (\theta_2 - Y_2)\Delta\bar{\theta}_2]\,d\tau, \tag{10f}$$

$$C_5 = \int_{\tau=0}^{\tau_f} [(\theta_1 - Y_1)\Delta\tilde{\theta}_1 + (\theta_2 - Y_2)\Delta\tilde{\theta}_2]\,d\tau. \tag{10g}$$

## 6. Adjoint problem and gradient equation

To obtain the adjoint problem, Eqs. (1a) and (1b) are multiplied by the Lagrange multiplier (or adjoint function) $\lambda_1(X,\tau)$ and $\lambda_2(X,\tau)$, respectively, and the resulting expression is integrated over the time and correspondent space domains. Then the result is added to the right hand side of Eq. (2) to yield the following expression for the functional $J[Bi_q(\tau), Bi_m(\tau)]$:

$$\begin{aligned} J[Bi_q(\tau), Bi_m(\tau)] &= \int_{X=0}^{1}\int_{\tau=0}^{\tau_f} [\theta_1(X,\tau) - Y_1(X,\tau)]^2 \delta(X - X_s)\,d\tau\,dX \\ &+ \int_{X=0}^{1}\int_{\tau=0}^{\tau_f} [\theta_2(X,\tau) - Y_2(X,\tau)]^2 \delta(X - X_s)\,d\tau\,dX \\ &+ \int_{X=0}^{1}\int_{\tau=0}^{\tau_f} \lambda_1(X,\tau)\left[\frac{\partial^2\theta_1(X,\tau)}{\partial X^2} - \varepsilon Ko\frac{\partial\theta_2(X,\tau)}{\partial\tau} - \frac{\partial\theta_1(X,\tau)}{\partial\tau}\right] d\tau\,dX + \int_{X=0}^{1}\int_{\tau=0}^{\tau_f} \lambda_2(X,\tau) \\ &\times \left[Lu\frac{\partial^2\theta_2(X,\tau)}{\partial X^2} - Lu\,Pn\frac{\partial^2\theta_1(X,\tau)}{\partial X^2} - \frac{\partial\theta_2(X,\tau)}{\partial\tau}\right] d\tau\,dX. \end{aligned} \tag{11}$$

Firstly, the variation $\Delta J_1$ is obtained by perturbing $Bi_q(\tau)$ by $Bi_q(\tau) + \Delta Bi_q(\tau)$, $\theta_1$ by $\theta_1(X,\tau) + \Delta\bar{\theta}_1$ and $\theta_2$ by $\theta_2 + \Delta\bar{\theta}_2$ in Eq. (11), subtracting from the resulting expression the original Eq. (11) and neglecting the second-order terms. We thus find

$$\begin{aligned} \Delta J_1[Bi_q(\tau), Bi_m(\tau)] &= \int_{X=0}^{1}\int_{\tau=0}^{\tau_f} 2[\theta_1(X,\tau) - Y_1(X,\tau)]\Delta\bar{\theta}_1\delta(X - X_s)\,d\tau\,dX \\ &+ \int_{X=0}^{1}\int_{\tau=0}^{\tau_f} 2[\theta_2(X,\tau) - Y_2(X,\tau)]\Delta\bar{\theta}_2\delta(X - X_s)\,d\tau\,dX \\ &+ \int_{X=0}^{1}\int_{\tau=0}^{\tau_f} \lambda_1(X,\tau)\left[\frac{\partial^2\Delta\bar{\theta}_1(X,\tau)}{\partial X^2} - \varepsilon Ko\frac{\partial\Delta\bar{\theta}_2(X,\tau)}{\partial\tau} - \frac{\partial\Delta\bar{\theta}_1(X,\tau)}{\partial\tau}\right] d\tau\,dX + \int_{X=0}^{1}\int_{\tau}^{\tau_f} \lambda_2(X,\tau) \\ &\times \left[Lu\frac{\partial^2\Delta\bar{\theta}_2(X,\tau)}{\partial X^2} - Lu\,Pn\frac{\partial^2\Delta\bar{\theta}_1(X,\tau)}{\partial X^2} - \frac{\partial\Delta\bar{\theta}_2(X,\tau)}{\partial\tau}\right] d\tau\,dX. \end{aligned} \tag{12}$$

In Eq. (12), the third and fourth integral terms are integrated by parts; the initial conditions of the sensitivity problem are utilized. The vanishing of the integrands leads to the following adjoint problem for the determination of $\lambda_1(X,\tau)$ and $\lambda_2(X,\tau)$:

$$\frac{\partial^2\lambda_1(X,\tau)}{\partial X^2} - Lu\,Pn\frac{\partial^2\lambda_2(X,\tau)}{\partial X^2} + 2(\theta_1 - Y_1)\delta(X - X_s) + \frac{\partial\lambda_1(X,\tau)}{\partial\tau} = 0, \quad \text{in } 0 \leqslant X \leqslant 1, \quad \tau > 0, \tag{13a}$$

$$Lu\frac{\partial^2\lambda_1(X,\tau)}{\partial X^2} + \varepsilon Ko\frac{\partial\lambda_1(X,\tau)}{\partial\tau} + 2(\theta_2 - Y_2)\delta(X - X_s) + \frac{\partial\lambda_2(X,\tau)}{\partial\tau} = 0, \quad \text{in } 0 \leqslant X \leqslant 1, \quad \tau > 0. \tag{13b}$$

Subjected to the following initial and boundary conditions

$$\lambda_1(X,\tau_f) = 0, \quad \text{in } 0 \leqslant \mathrm{X} \leqslant 1,\ \tau = \tau_f, \tag{13c}$$

$$\lambda_2(X,\tau_f) + \varepsilon Ko\lambda_1(X,\tau_f) = 0, \quad \text{in } 0 \leqslant X \leqslant 1,\ \tau = \tau_f, \tag{13d}$$

$$\frac{\partial\lambda_1(X,\tau)}{\partial X} = \mathrm{LuPn}\frac{\partial\lambda_2(X,\tau)}{\partial X}, \quad \text{at } X = 0,\ \tau > 0, \tag{13e}$$

$$\frac{\partial\theta_2(X,\tau)}{\partial X} = 0, \quad \text{at } X = 0,\ \tau > 0, \tag{13f}$$

$$Lu\,Pn\frac{\partial\lambda_2(X,\tau)}{\partial X} - \frac{\partial\lambda_1(X,\tau)}{\partial X} - Bi_q(\tau)\lambda_1(X,\tau) = 0, \quad \text{at } X = 1,\ \tau > 0, \tag{13g}$$

$$-Lu\frac{\partial\lambda_2(X,\tau)}{\partial X} - Lu\lambda_2 Bi_m(\tau) + (1-\varepsilon)Ko\,Lu\,Bi_m(\tau) \times \lambda_1(X,\tau) = 0, \quad \text{at } X = 1,\ \tau > 0. \tag{13h}$$

It is obvious that the adjoint problem is a one-layer not a two-layer problem when interior measurements are utilized. The adjoint problems are different from the standard initial value problems in that the final time conditions at time $\tau = \tau_f$ is specified instead of the customary initial condition. However, this problem can be transformed to an initial value problem by the transformation of the time variables as $\tau^* = \tau_f - \tau$. Then the techniques of Crank–Nicolson finite differences method with iterations can be used to solve the above adjoint problems.

Finally, the following integral term is left

$$\Delta J_1 = \int_{\tau=0}^{\tau_f} \lambda_1(1-\theta_1)\Delta Bi_q \, d\tau. \tag{14}$$

From definition [1], the functional increment can be presented as

$$\Delta J_1 = \int_{\tau=0}^{\tau_f} (J_1' \Delta Bi_q)\, dt. \tag{15}$$

A comparison of Eqs. (14) and (15) leads to the following expression for the gradient of functional $J_1'$:

$$J_1'[Bi_q(\tau)] = \lambda_1(1,\tau)[1-\theta_1(1,\tau)]. \tag{16}$$

Similarly, to derive the adjoint problems for the case when perturbing $Bi_m(\tau)$, Eqs. (1a) and (1b) are multiplied by the Lagrange multiplier (or adjoint function) $\lambda_3(X,\tau)$ and $\lambda_4(X,\tau)$ and follow the same procedure as described previously. Eventually we find that the solutions for adjoint equation of $\lambda_3(X,\tau)$ and $\lambda_((X,\tau)$ are identical to that for $\lambda_1(X,\tau)$ and $\lambda_2(X,\tau)$. This implies that the adjoint equations need to be solved only once since $\lambda_1(X,\tau) = \lambda_3(X,\tau)$ and $\lambda_2(X,\tau) = \lambda_4(X,\tau)$. Finally the gradient equation for $Bi_m(\tau)$ can be obtained as

$$\begin{aligned} J_2'[Bi_m(\tau)] = &-Ko\,Lu\lambda_1(1,\tau) \times [1-\theta_2(1,\tau)] \\ &+ Lu\,\lambda_2(1,\tau) \times [1-\theta_2(1,\tau)]. \end{aligned} \tag{17}$$

We note that $J_1'[Bi_q(\tau_f)]$ and $J_2'[Bi_m(\tau_f)]$ are always equal to zero since $\lambda_1(1,\tau_f) = \lambda_2(1,\tau_f) = 0$ at $\tau = \tau_f$. With this fact and Eqs. (3a)–(5b) we concluded that the estimated values for $Bi_m(\tau_f)$ and $Bi_q(\tau_f)$ are definitely equal to the values of its initial guess. We will show this by using numerical experiments in the section of results and discussions and then raise a question about this point for Ref. [5].

One easy way to improve the prediction at end time $\tau_f$ is to extend the measurement time. For instant, if end time $\tau_f = 10$, we should measure the data up to, say, $\tau = 12$ and then perform the inverse calculations. Finally extract the inverse solutions to $\tau_f = 10$. The singularity near $\tau_f$ can greatly be improved.

## 7. Stopping criterion

If the problem contains no measurement errors, the traditional check condition is specified as

$$J[Bi_q(\tau), Bi_m(\tau)] < \eta, \tag{18}$$

where $\eta$ is a small-specified number. However, the measured temperature and moisture data may contain measurement errors. Therefore, we do not expect the functional Eq. (2) to be equal to zero at the final iteration step. Following the experience of the authors [1–4], we use the discrepancy principle as the stopping criterion, i.e. we assume that the residuals for temperature and moisture may be approximated by

$$\theta_1(X,\tau) - Y_1(X,\tau) = \theta_2(X,\tau) - Y_2(X,\tau) \approx \sigma, \tag{19}$$

where $\sigma$ is the stand deviation of the measurements, which is assumed to be a constant.

Substituting Eq. (19) into Eq. (2), the following expression is obtained for $\eta$:

$$\eta = \sigma^2 \tau_f. \tag{20}$$

Then, the stopping criterion is given by Eq. (18) with $\eta$ determined from Eq. (20).

## 8. Computational procedure

The computational procedure for the solution of this inverse heat and mass transfer problem may be summarized as follows:

Suppose $Bi_q^n(\tau)$ and $Bi_m^n(\tau)$ are available at iteration $n$.

Step 1. Solve the direct nonlinear problem given by Eqs. (1a)–(1h) for $\theta_1(X,\tau)$ and $\theta_2(X,\tau)$.
Step 2. Examine the stopping criterion $\varepsilon$. Continue if not satisfied.
Step 3. Solve the adjoint problem given by Eqs. (13a)–(13h) for $\lambda_1(X,\tau)$ and $\lambda_2(X,\tau)$.
Step 4. Compute the gradient of the functional $J_1'[Bi_q(\tau)]$ and $J_2'[Bi_m(\tau)]$ from Eqs. (16) and (17), respectively.
Step 5. Compute the conjugate coefficients $\gamma_1^n$ and $\gamma_2^n$ and the direction of descent $P_1^n(\tau)$ and $P_2^n(\tau)$ from Eqs. (5a) and (5b) and Eqs. (4a) and (4b), respectively.
Step 6. Set $\Delta Bi_q(\tau) = P_1^n(\tau)$ and $\Delta Bi_m(\tau) = P_2^n(\tau)$, and solve the sensitivity problems given by Eqs. (6a)–(7h) for $\Delta\bar{\theta}_1(P_1^n)$, $\Delta\bar{\theta}_2(P_1^n)$ and $\Delta\tilde{\theta}_1(P_2^n)$, $\Delta\tilde{\theta}_2(P_2^n)$.
Step 7. Compute the search step sizes $\beta_1^n$ and $\beta_2^n$ from Eqs. (10a)–(10g).
Step 8. Compute the new estimation for $Bi_q^{n+1}(\tau)$ and $Bi_m^{n+1}(\tau)$ from Eqs. (3a) and (3b) and return to step 1.

## 9. Results and discussions

The objective of this article is to show the validity of the CGM in simultaneous estimating the time-dependent Biot numbers $Bi_q(\tau)$ and $Bi_m(\tau)$ for heat and moisture transfer problem with no prior information on the functional form of the unknown quantities.

Before proceed to study the inverse problem that we are going to consider here, one should make sure first that the numerical solution for the direct problem is correct, otherwise the discussions of the inverse solutions will become meaningless. To test the accuracy of the direct problem we first solve Eqs. (1a)–(1h) by using the following given conditions and quantities:

$Lu = 0.02,\ \ \varepsilon = 0.2,\ \ Bi_m = 2.5,\ \ Bi_q = 2.5,$

$Ko = 5.0,\ \ Pn = 0.6,\ \ Q = 0.9.$

Besides, the space and time increments used in numerical calculations are taken as $\Delta X = 0.01$ (i.e. 100 grid points in space) and $\Delta\tau = 0.001$, respectively. The comparison of analytical [6] and numerical temperature and moisture distributions are shown in Figs. 2 and 3, respectively. We should note that the original figure (i.e. Fig. 7) for temperature distribution in Ref. [6] may exist some mistype errors for Fourier number (or dimensionless time), the indicated $\tau$ (or $Fo$) $= 0.1, 0.2, 0.4, 0.8, 1.6$ and $3.2$ should be replaced by $\tau$ (or $Fo$) $= 0.05, 0.1, 0.2, 0.4, 0.8$ and $1.6$, while $\tau = 3.2$ was not reported in the original figure in Ref. [6]. For this reason we did not show the analytical solution for $\tau = 3.2$ in Fig. 2. It can be seen from Figs. 2 and 3 that they are in a good agreement, therefore the verification of our numerical program for direct problem is thus completed.

To illustrate the accuracy of the CGM in simultaneous predicting Biot numbers $Bi_q(\tau)$ and $Bi_m(\tau)$ with inverse heat and mass transfer analysis from the knowledge of measured transient temperature and moisture distributions, we consider two specific examples where the Biot numbers $Bi_q(\tau)$ and $Bi_m(\tau)$ are in different forms.

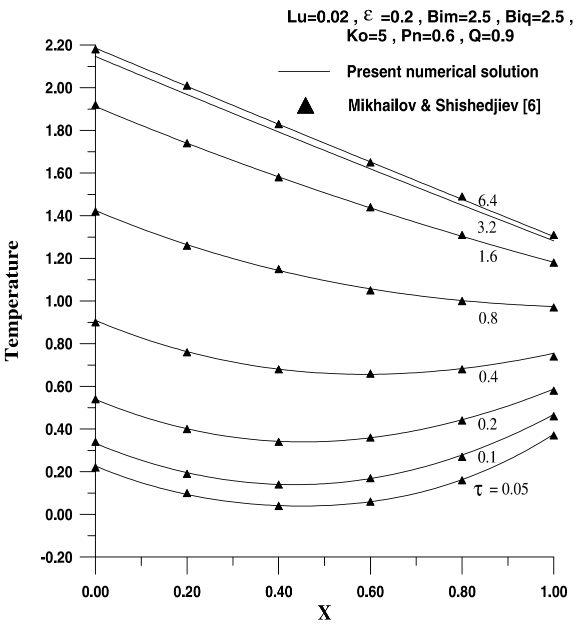


Fig. 2. The comparison of analytical and numerical solution for temperature distributions.

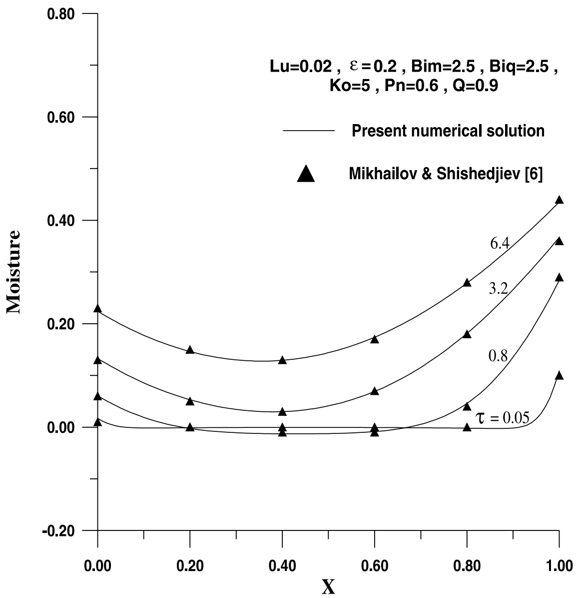


Fig. 3. The comparison of analytical and numerical solution for moisture distributions.

One of the advantages of using the CGM is that the initial guesses of the unknown Biot numbers $Bi_q(\tau)$ and $Bi_m(\tau)$ can be chosen arbitrarily. In all the test cases considered here, the initial guesses of $Bi_q(\tau)$ and $Bi_m(\tau)$ used to begin the iteration are taken as $Bi_q^0(\tau) = Bi_m^0(\tau) = 0.0$.

In order to compare the results for situations involving random measurement errors, we assume normally distributed uncorrelated errors with zero mean and constant standard deviation. The simulated inexact measurement data $Y$ can be expressed as

$$Y = Y_{exact} + \omega\sigma, \tag{21}$$

where $Y_{exact}$ is the solution of the direct problem with an exact $Bi_q(\tau)$ and $Bi_m(\tau)$; $\sigma$ is the standard deviation of the measurements; and $\omega$ is a random variable that generated by subroutine DRNNOR of the IMSL [9] and will be within $-2.576$ to $2.576$ for a 99% confidence bound.

We now present below two numerical experiments in determining $Bi_q(\tau)$ and $Bi_m(\tau)$ by the inverse analysis.

### 9.1. Numerical test case 1

The parameters for the direct problem are given as follows:

$Lu = 0.4,\ \ \varepsilon = 0.2,\ \ Ko = 5.0,\ \ Pn = 0.6,\ \ Q = 0.0.$

Besides, the space and time increments used in numerical calculations are taken as $\Delta X = 0.02$ for $X = 1.0$ (i.e. $m = 1$–$50$ and $m$ is the grid points in space) and $\Delta\tau = 0.002$ for total time $\tau_f = 1.0$ (i.e. 500 discreted time), respectively. Therefore a total of 1000 unknown discreted Biot numbers are to be determined in the present study.

The exact time-dependent Biot numbers $Bi_q(\tau)$ and $Bi_m(\tau)$ for heat and moisture transfer problem are assumed as

$$Bi_q(\tau) = 5 - 2\cos\left(4\pi\frac{\tau}{\tau_f}\right), \quad 0 < \tau \leqslant \tau_f, \tag{22a}$$

$$Bi_m(\tau) = 3 + 2\sin\left(4\pi\frac{\tau}{\tau_f}\right), \quad 0 < \tau \leqslant \tau_f. \tag{22b}$$

One should note that in the present test case we use initial guess $Bi_q^0(\tau) = Bi_m^0(\tau) = 0.0$, but now the exact values of $Bi_q(\tau_f)$ and $Bi_m(\tau_f)$ are not equal to zero, therefore we concluded that the singularity at final time $\tau_f$ will be happen in this case and the estimated values for $Bi_q(\tau_f)$ and $Bi_m(\tau_f)$ must be the same as the initial guess values, i.e. $Bi_q(\tau_f) = Bi_m(\tau_f) = 0.0$.

The inverse analysis is first performed by assuming exact measurements, $\sigma = 0.0$ and by using measurement data at measure position $m = 48$ (i.e. at $X_s = 0.96$ and very close to the boundary). By setting 30 iterations, the functional can be decreased to $J = 6.12 \times 10^{-6}$. The measured and estimated temperatures ($Y_1$ and $\theta_1$) and moistures ($Y_2$ and $\theta_2$) are shown in Fig. 4 while the exact

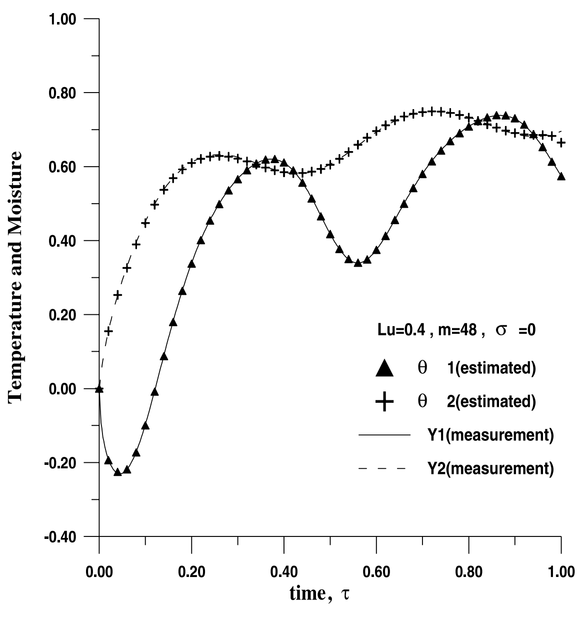


Fig. 4. The measured and estimated temperature and moisture distributions with $\sigma = 0.0$ at $m = 48$ in test case 1.

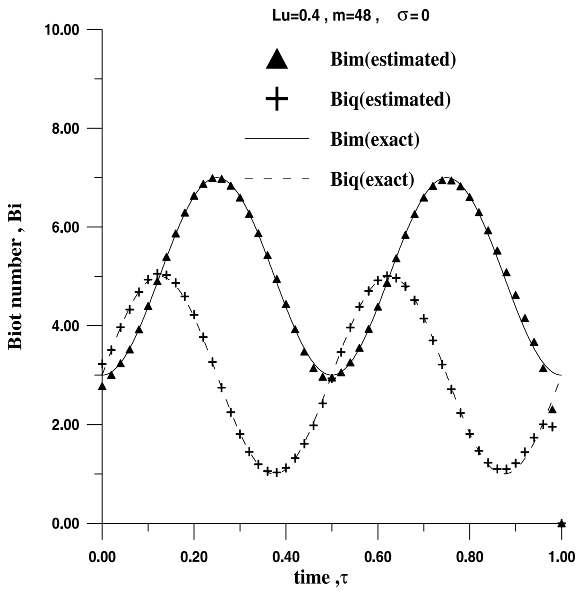


Fig. 5. The exact and estimated Biot number distributions with $\sigma = 0.0$ at $m = 48$ in test case 1.

and estimated $Bi_q(\tau)$ and $Bi_m(\tau)$ are shown in Fig. 5. It can be seen from Fig. 4 that there is a good agreement between the measured and estimated temperatures and moistures. Moreover, form Fig. 5 we learn that the estimations for $Bi_q(\tau)$ and $Bi_m(\tau)$ are very accurate except near final time. The estimated $Bi_q(\tau_f)$ and $Bi_m(\tau_f)$ at final time $\tau_f$ both approach to zero, (i.e. the initial guess value) due to the singularity addressed previously.

This phenomena should but not be seen in Ref. [5]. In their paper the initial guess of moisture flux is taken as 0.4 and $10^{-3}$, respectively, but none of the estimated moisture flux approaches to the initial guess values at final time $\tau_f$, those results are suspicious.

The average errors for $Bi_q(\tau)$ and $Bi_m(\tau)$ are calculated as ERR1 = 0.43% and ERR2 = 1.13%, respectively, where the average errors for the estimated $Bi_q(\tau)$ and $Bi_m(\tau)$ are defined as

$$\mathrm{ERR1}\% = \left[\sum_{J=1}^{500}\left|\frac{Bi_q(J) - \widehat{Bi}_q(J)}{Bi_q(J)}\right|\right] \div 500 \times 100\%, \tag{23a}$$

$$\mathrm{ERR2}\% = \left[\sum_{J=1}^{500}\left|\frac{Bi_m(J) - \widehat{Bi}_m(J)}{Bi_m(J)}\right|\right] \div 500 \times 100\%. \tag{23b}$$

Here $J$ represents the index of discreted time, while $Bi_q(J)$ and $Bi_m(J)$ denote the exact values while $\widehat{Bi}_q(J)$ and $\widehat{Bi}_m(J)$ denote the estimated values of Biot number.

Then the inverse analysis is performed again by using measurement data measured at $m = 40$ (i.e. $X_s = 0.8$).

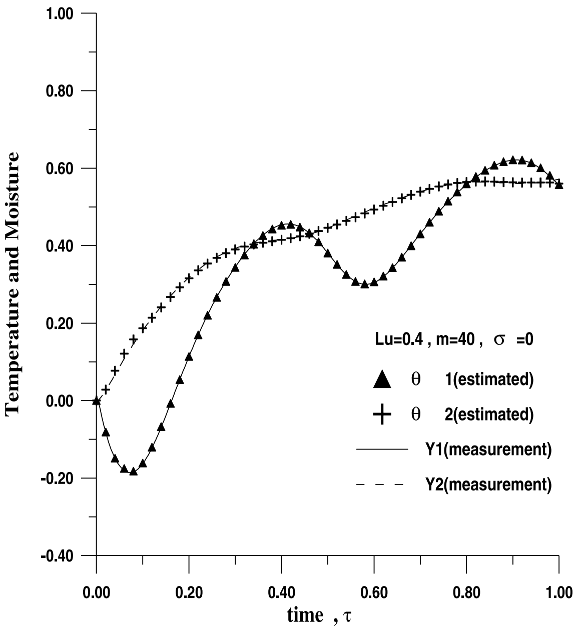


Fig. 6. The measured and estimated temperature and moisture distributions with $\sigma = 0.0$ at $m = 40$ in test case 1.

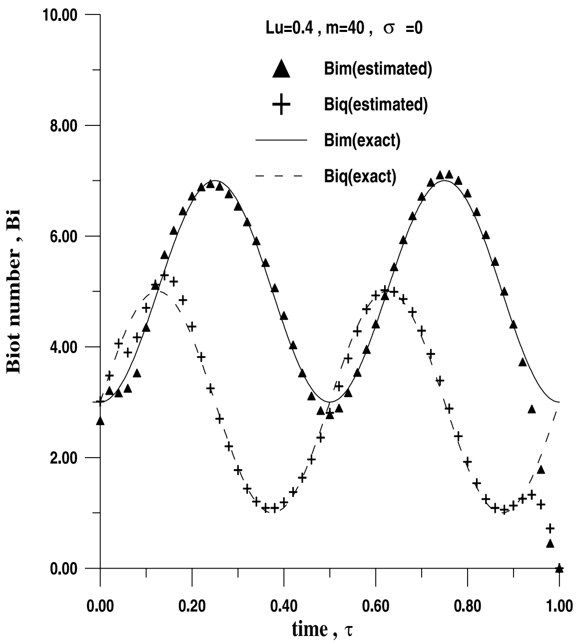


Fig. 7. The exact and estimated Biot number distributions with $\sigma = 0.0$ at $m = 40$ in test case 1.

By setting 30 iterations, the functional can be decreased to $J = 4.61 \times 10^{-6}$. The measured and estimated temperatures ($Y_1$ and $\theta_1$) and moistures ($Y_2$ and $\theta_2$) are also in a good agreement and is shown in Fig. 6. However, the accuracy of the estimated values of Biot number is decreased and the results are shown in Fig. 7. The average errors for $Bi_q(\tau)$ and $Bi_m(\tau)$ are calculated as ERR1 = 2.66% and ERR2 = 3.83%, respectively.

Next, let us discuss the influence of the measurement errors on the inverse solutions. First, the measurement error for the temperature and moisture measured at $m = 48$ is taken as $\sigma = 0.005$, then error is increased to $\sigma = 0.02$. The estimated $Bi_q(\tau)$ and $Bi_m(\tau)$ is shown in Figs. 8 and 9, respectively. The stopping criteria can be obtained by discrepancy principle and given in Eq. (20). The number of iteration for $\sigma = 0.005$ is 18 and the average errors for $Bi_q(\tau)$ and $Bi_m(\tau)$ are calculated as ERR1 = 1.07% and ERR2 = 1.42%, respectively. The number of iteration for $\sigma = 0.02$ is 10 and the average errors for $Bi_q(\tau)$ and $Bi_m(\tau)$ are calculated as ERR1 = 5.13% and ERR2 = 9.13%, respectively. This implies that reliable inverse solutions can still be obtained when measurement errors are considered.

### 9.2. Numerical test case 2

The parameters for the direct problem are given as follows:

$$Lu = 0.02,\ \ \varepsilon = 0.2,\ \ Ko = 5.0,\ \ Pn = 0.6,\ \ Q = 0.0.$$

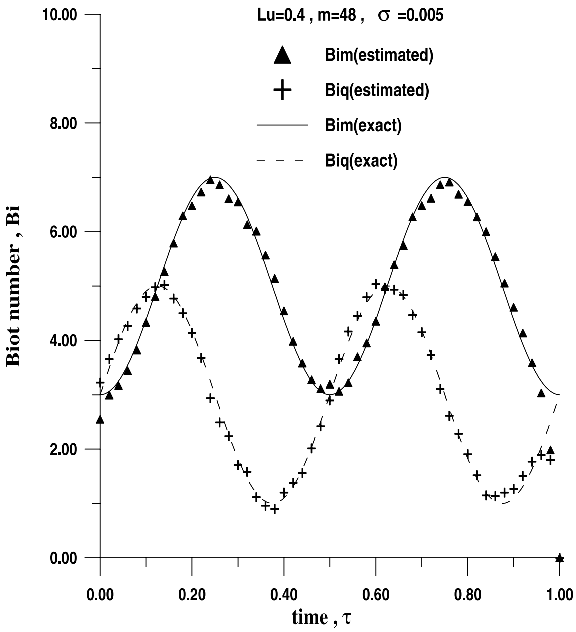


Fig. 8. The exact and estimated Biot number distributions with $\sigma = 0.05$ at $m = 48$ in test case 1.

Besides, the space and time increments used in numerical calculations are taken the same as were used in numerical test case 1.

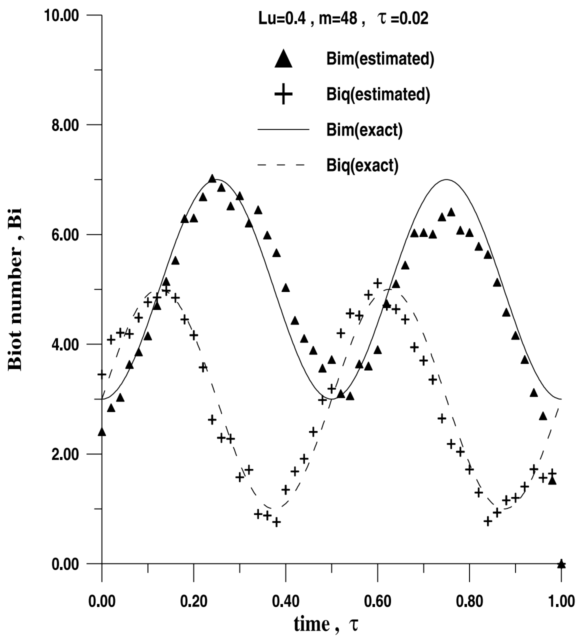


Fig. 9. The exact and estimated Biot number distributions with $\sigma = 0.002$ at $m = 48$ in test case 1.

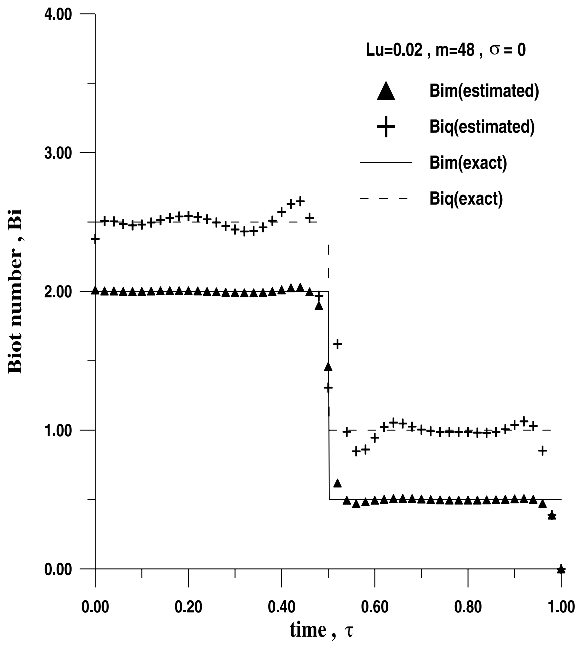


Fig. 10. The exact and estimated Biot number distributions with $\sigma = 0.0$ at $m = 48$ in test case 2.

The exact time-dependent Biot numbers $Bi_q(\tau)$ and $Bi_m(\tau)$ are now assumed as a step function and given as follows

$$\begin{cases} Bi_q(\tau) = 2.0, & 0 < \tau \leqslant 0.5, \\ Bi_q(\tau) = 0.5, & 0.5 < \tau \leqslant \tau_f, \end{cases} \tag{24a}$$

$$\begin{cases} Bi_m(\tau) = 2.5, & 0 < \tau \leqslant 0.5, \\ Bi_m(\tau) = 1.0, & 0.5 < \tau \leqslant \tau_f. \end{cases} \tag{24b}$$

Test case 2 is a more rigorous examination since there is a discontinuity for Biot numbers. It is expected that the inverse solutions are worse than test case 1.

The inverse analysis is first performed by assuming exact measurements, $\sigma = 0.0$ and by using measurement data at measure position $m = 48$ (i.e. at $X_s = 0.96$). By setting 30 iterations, the functional can be decreased to $J = 3.03 \times 10^{-7}$. The exact and estimated $Bi_q(\tau)$ and $Bi_m(\tau)$ are shown in Fig. 10. The estimated $Bi_q(\tau_f)$ and $Bi_m(\tau_f)$ at final time $\tau_f$ also approach to zero for the reason stated previously. The average errors for $Bi_q(\tau)$ and $Bi_m(\tau)$ are determined as $\mathrm{ERR1} = 0.47\%$ and $\mathrm{ERR2} = 1.38\%$, respectively.

Then the influence of the measurement errors on the inverse solutions is examined. The measurement error for the temperature and moisture measured at $m = 48$ is firstly taken as $\sigma = 0.001$, the stopping criteria can be obtained from Eq. (20) and the number of iteration is 22. The estimated $Bi_q(\tau)$ and $Bi_m(\tau)$ is shown in Fig. 11 and the average errors for $Bi_q(\tau)$ and $Bi_m(\tau)$ are calculated as $\mathrm{ERR1} = 0.52\%$ and $\mathrm{ERR2} = 1.51\%$, respectively. Then error is increased to $\sigma = 0.005$. After 10

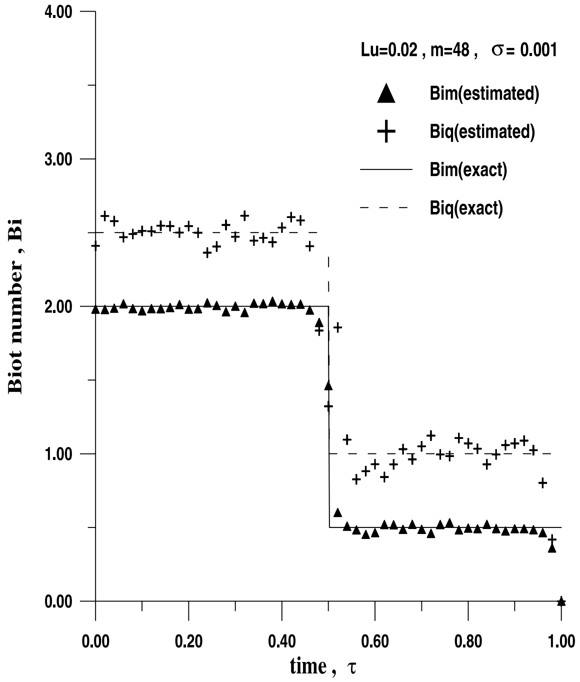


Fig. 11. The exact and estimated Biot number distributions with $\sigma = 0.001$ at $m = 48$ in test case 2.

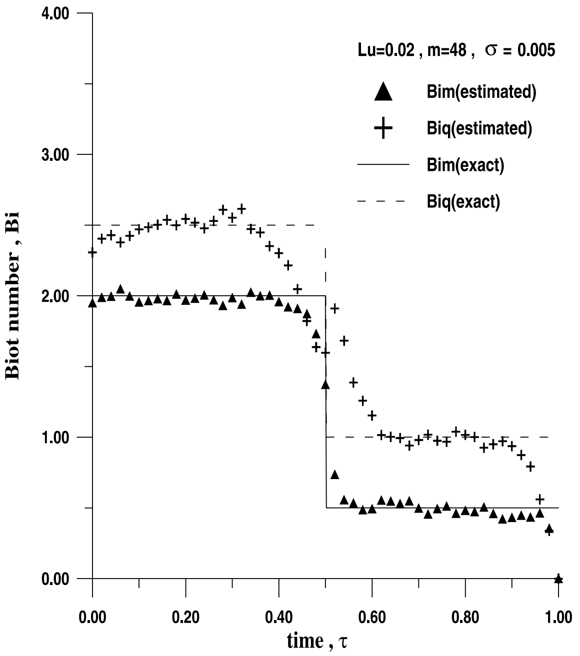


Fig. 12. The exact and estimated Biot number distributions with $\sigma = 0.005$ at $m = 48$ in test case 2.

iterations the inverse solution can be obtained and the result is shown in Fig. 12. The average errors for $Bi_q(\tau)$ and $Bi_m(\tau)$ are calculated as $\mathrm{ERR1} = 0.86\%$ and $\mathrm{ERR2} = 2.03\%$, respectively.

From the above two test cases we learned that an inverse heat and moisture transfer problem in simultaneous estimating the time-dependent Biot numbers $Bi_q(\tau)$ and $Bi_m(\tau)$ for a porous material is now completed. Reliable estimations can be obtained when using either exact or error measurements.

## 10. Conclusions

The CGM was successfully applied for the solution of the inverse heat and mass transfer conduction problem to estimate the unknown the time-dependent Biot numbers $Bi_q(\tau)$ and $Bi_m(\tau)$ for a porous material by utilizing simulated temperature readings obtained from different measured positions.

Two test cases involving different form of Biot numbers and measurement errors were considered. The results show that the inverse solutions obtained by CGM are still very accurate as the measurement errors are increased.

## Acknowledgements

This work was supported in part through the National Science Council, R.O.C., Grant number, NSC-90-2611-E-006-012.

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